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Glint Errors Derived From the Partial Derivatives of the Echo Signal Phase for a Distributed Scatterer

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13 ABSTRACT *(Maximum 200 words)*

This report extends the study of radar measurements in [1] from a point scatterer to a distributed scatterer. The measurement is based on partial derivatives of the echo-signal phase The study is aimed at developing an understanding of what kind of radar information can be measured from echo-signal and formulating glint errors caused by a distributed scatterer

The importance of the phase has been shown in signal and image processing Basic radar measurements can also be obtained from the echo-signal phase [1,2,3] Unlike measuring of target information from a point scatterer, for a distributed scatterer we must take account of the measurement errors caused by the dimension and the orientation of a distributed scatterer These are called glint errors In this report, we describe how to measure radar information from the partial derivatives of the echo-signal phase with respect to frequency, time and spatial position We derive the expressions of range, range-rate, angle, and angle-rate glint errors for a distributed scatterer, and give an insight into these glint errors

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GLINT ERRORS DERIVED FROM THE PARTIAL DERIVATIVES OF THE ECHO SIGNAL PHASE FOR A DISTRIBUTED SCATTERER

1. Introduction

From an information transmission point of view, the information carriers in a time-varying signal are amplitude, frequency, and phase. Because of the relationship between frequency and phase the time-derivative of phase is frequency and time-integral of frequency is phase. The information carried by frequency is also available in the phase. Therefore, we are only concerned with the information in amplitude and phase.

A time-varying signal is represented in the time-domain by:

$$s(t) = a(t)\, e^{j\Phi(t)},$$

where $a(t)$ is the amplitude and $\Phi(t)$ is the phase (or temporal phase):

$$\Phi(t) = 2\pi f_c\, t + \Phi_0(t),$$

where f_c is the signal carrier frequency and $\Phi_0(t)$ is the time-varying phase.

A time-varying signal can also be represented in the frequency domain by taking the Fourier transformation. In the frequency domain, the information is carried by the amplitude spectrum $\tilde{a}(f)$ and the phase spectrum $\tilde{\Phi}(f)$.

As described in [1],[2], and [3], the information which can be extracted from a point scaterer are

$$\frac{\partial \tilde{\Phi}(f)}{\partial f}\Big|_t \; - \; \textit{Range information}$$

$$\frac{\partial \Phi(t)}{\partial t}\Big|_f \; - \; \textit{Range-rate}$$

$$\frac{\partial \tilde{a}(f)}{\partial f}\Big|_t \; - \; \textit{Radial size}$$

$$\frac{\partial a(t)}{\partial t}\Big|_f \; - \; \textit{Change of radial profile}$$

Angular information can also be extracted from echo signals by use of more than one antenna element; for example, a simple interferometer with two antenna elements separated

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by d described in [1]. The phase difference $\Phi_{12}(d)$ or the spectrum of the phase difference $\tilde{\Phi}_{12}(f)$ between the two signals received at antenna element 1 and element 2, called spatial phase or spatial phase spectrum, is needed to extract angle and angle-rate. With spatial antenna elements, the information which can be extracted is

$$\frac{\partial \Phi_{12}(d)}{\partial d}\Big|_{f,t} \rightarrow \textit{Angular information}$$

$$\frac{\partial \tilde{\Phi}_{12}(f)}{\partial f}\Big|_{d,t} \rightarrow \textit{Angular information}$$

$$\frac{\partial \Phi_{12}(d)}{\partial t}\Big|_{d,f},\ \frac{\partial \tilde{\Phi}_{12}(f)}{\partial t}\Big|_{d,f} \rightarrow \textit{Angular rate}$$

$$\frac{\partial a_r(d)}{\partial d}\Big|_{f,t} \rightarrow \textit{Tangential profile information}$$

where $a_r(d)$ is the amplitude difference between the two signals received at antenna element 1 and element 2.

The amplitude of the radar echo signal also provides information about the target size, shape and change of shape. The importance of the phase has been shown in signal and image processing [4]. Phase also plays an important role in extracting radar information as described in this report. It can provide the measurement of range, range-rate, angle, and angle-rate. In this report, we deal with information extraction from phase in time-domain or phase spectrum in Fourier-domain.

When the maximum dimension of a target is greater than the dimension of the radar resolution cell, the target is no longer a point scatterer and it should be modeled as a distributed scatterer. Unlike the measuring of target information from a point scatterer, for distributed scatterers we must take account of the measurement errors caused by the target dimension and orientation, which are called glint errors. Glint error results in the wandering of the apparent center of the target.

A distributed scatterer can be modeled as N point scatterers as shown in Fig.1. The dimension of each point scatterer is within the radar resolution cell [5]. With this model the radar returned signal from the distributed scatterer can be calculated as the sum of the returned signal from each point scatterer.

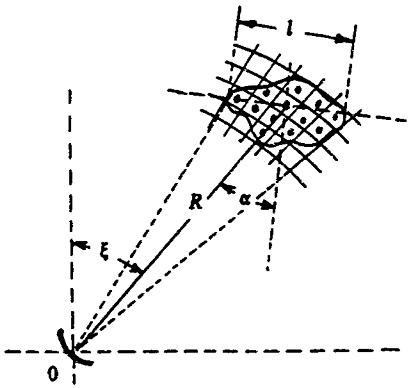

Fig.1 A distributed scatterer.

In the report, we only deal with range profiles, based on a time-varying temporal signal from a given direction as illustrated in Fig.2.

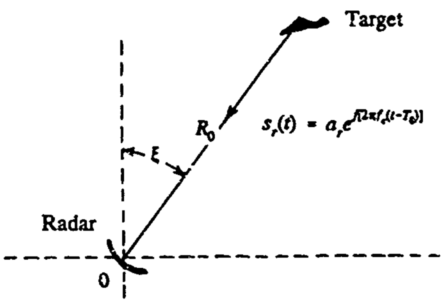

Fig.2 Radar returned time-varying temporal signal.

Assuming that the transmitted signal is a sinusoidal wave of frequency f_c, represented as

$$s_t(t) = a_t\, e^{j2\pi f_c t},$$

where a_t is the amplitude of the transmitted signal, and the phase is taken to be zero, then, the returned signal from the distributed N-point scatterer is the summation of the returned signals from each point scatterer

$$s_r(t) = \sum_{i=1}^{N} a_{ri}\, s_t(t-T_i), \tag{1}$$

where a_{ri} is the amplitude and T_i is the time delay of the returned signal from the ith point scatterer.

Let a_r be the ampiitude and Φ_r be the phase of the sum signal $s_r(t)$:

$$s_r(t) = a_r\, e^{j(2\pi f_c t+\Phi_r)},$$

where the phase Φ_r consists of two parts: one is due to a time delay from the target geometric center $2\pi f_c T_0$; the other is a phase offset Φ due to the dimension and the orientation of the distributed scatterer. We have

$$\Phi_r = -2\pi f_c T_0 + \Phi. \tag{2}$$

On the other hand, the phase shift of the returned signal from the ith point scatterer $2\pi f_c T_i$ also consists of two parts: one is due to a time delay from the target geometric center, $2\pi f_c T_0$, and the other is a phase offset from the target geometric center, Ψ_i. Therefore, we can rewrite Eq.(1) as

$$a_r e^{j(2\pi f_c t - 2\pi f_c T_0 + \Phi)} = \sum_{i=1}^{N} a_{ri}\, e^{j(2\pi f_c t - 2\pi f_c T_0 + \Psi_i)}$$

or

$$a_r e^{j\Phi} = \sum_{i=1}^{N} a_{ri}\, e^{j\Psi_i}. \tag{3}$$

Let us separate the complex exponential function in Eq.(3) into a real part and an imaginary part. Then Eq.(3) becomes

$$a_r \cos\Phi = \sum_{i=1}^{N} a_{ri} \cos\Psi_i, \tag{4}$$

$$a_r \sin\Phi = \sum_{i=1}^{N} a_{ri} \sin\Psi_i. \tag{5}$$

From Eq.(4) and (5), the phase offset Φ in the sum signal $s_r(t)$ can be obtained as

$$\Phi = \arctan\frac{\sum_i a_{ri}\sin\Psi_i}{\sum_i a_{ri}\cos\Psi_i}. \tag{6}$$

Phase Φ_r in Eq.(2) is called the <u>temporal phase</u>. The temporal phase can be represented in terms of the target range R_0. If the target is moving with radial velocity V_r, the temporal phase is

$$\Phi_r = 2\pi f_c \frac{2(R_0 - V_r t)}{c} + \Phi, \tag{7}$$

where c is velocity of propagation. The temporal phase spectrum in the Fourier domain is

$$\bar{\Phi}_r = 2\pi f T_0 + \Phi.$$

Similar to the analysis for a point scatterer in [1], for the case of a distributed scatterer the partial derivatives of the temporal phase can also provide range, range-rate, angle, and angle-rate information along with corresponding glint errors.

The following analysis will describe measurements of range, range-rate, angle and angle-rate, and will derive the corresponding glint errors. The range and angle glint errors derived here are identical to those results stated in [5]. Exact expressions for range and angle glint errors can be found in [6].

2. Range Measurement and Range Glint Error

In the case of a distributed scatterer, the range information can be extracted from the partial derivative of the temporal phase with respect to frequency. To resolve range ambiguities, we must use multiple frequencies: $f_1, f_2, \ldots f_M$. Taking the partial derivative of the temporal phase spectrum with respect to frequency f, we obtain target range information as the following:

$$R = \frac{c}{4\pi}\left[\frac{\partial\tilde{\Phi}_r}{\partial f} - \frac{\partial\Phi}{\partial f}\right], \tag{8}$$

where the second term represents the range glint error which causes inaccurate range measurement. The range glint error is determined by

$$\Delta R = \frac{c}{4\pi}\frac{\partial\Phi}{\partial f}. \tag{9}$$

To analyze the range glint error let us use the simplest two-point scatterer as an example as shown in Fig.3.

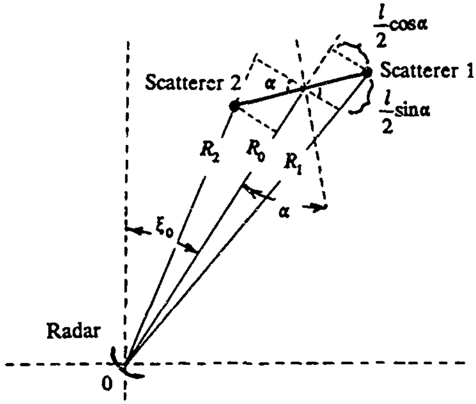

Fig.3 A two-point scatterer.

Assuming the size of the distributed scatterer is l, the orientation of the target is α, the range from radar to the target geometric center is R_0, the range from radar to the point-scatterer 1 is R_1, and from radar to the point-scatterer 2 is R_2, then from Fig.3 we know that the range difference between point 1 and point 2 is $l \sin\alpha$. For the signal returned from point 1, the phase offset from the target geometric center is

$$\Psi_1 = 2\pi f_c \frac{l \sin\alpha}{c} = \frac{2\pi}{\lambda} l \sin\alpha. \tag{10}$$

In the signal returned from the point scatterer 2, the phase offset from that of target geometric center is

$$\Psi_2 = -2\pi f_c \frac{l \sin\alpha}{c} = -\frac{2\pi}{\lambda} l \sin\alpha. \tag{11}$$

If the phase difference ϕ_1, relative to the phase at the target geometric center and returned from the point scatterer 1, is different from the phase difference ϕ_2 from the point scatterer 2, we should add phase differences ϕ_1 and ϕ_2 to the phase offset in Eq.(10) and (11), respectively. Therefore, the above two phase offsets become :

$$\Psi_1 = 2\pi f_c \frac{l \sin\alpha}{c} + \phi_1 = \frac{2\pi}{\lambda} l \sin\alpha + \phi_1, \tag{12}$$

and

$$\Psi_2 = -2\pi f_c \frac{l \sin\alpha}{c} + \phi_2 = -\frac{2\pi}{\lambda} l \sin\alpha + \phi_2. \tag{13}$$

For the case of a two-point scatterer, Eqs.(4) and (5) are reduced to

$$a_r \cos\Phi = a_{r1} \cos\Psi_1 + a_{r2} \cos\Psi_2, \tag{14}$$

and

$$a_r \sin\Phi = a_{r1} \sin\Psi_1 + a_{r2} \sin\Psi_2. \tag{15}$$

Then the phase offset becomes

$$\Phi = \arctan\frac{a_{r1} \sin\Psi_1 + a_{r2} \sin\Psi_2}{a_{r2} \cos\Psi_1 + a_{r2} \cos\Psi_2}. \tag{16}$$

Taking the partial derivative of the phase offset with respect to frequency, we have

$$\frac{\partial\Phi}{\partial f} = \frac{2\pi l \sin\alpha}{c} \frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2 + a_{r2}^2 + 2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha + \delta\phi)}, \tag{17}$$

where $\delta\phi = \phi_1 - \phi_2$. Finally, the range glint error can be obtained as

$$\Delta R = \frac{c}{4\pi}\frac{\partial\Phi}{\partial f} = \frac{l \sin\alpha}{2} \frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2 + 2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda} \sin\alpha + \delta\phi) + a_{r2}^2}, \tag{18}$$

where $l \sin\alpha$ is the radial projection of the target length l.

Assuming $\delta\phi = 0$, we can see from Eq.(18) that

(a) if $a_{r1} = 0$, then $\Delta R = (l \sin\alpha)/2$: the range glint error is induced by the point 2 and the radar is tracking the range of point 2;

(b) if $a_{r2} = 0$, then $\Delta R = -(l \sin\alpha)/2$: the range glint error is induced by the point 1 and the radar is tracking the range of point 1;

(c) if $a_{r1} = a_{r2}$ and $\cos(4\pi l \sin\alpha/\lambda) \neq -1$, then $\Delta R = 0$: the radar is tracking the target geometric center;

(d) if $a_{r1} = a_{r2}$ and $\cos(4\pi l \sin\alpha/\lambda) = -1$, then $\Delta R = \pm\infty$: the radar is not able to track the target at all.

Fig.4 shows the range glint error varying with the target orientation angle α, where we assume $f = 1\ (GHz)$ and $l = 10\ (m)$.

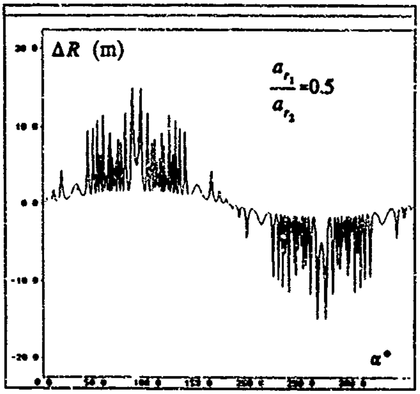

Fig.4 The range glint error varies with target orientation angles.

3. Range-Rate Measurement and Range-Rate Glint Error

The range-rate information can be extracted from the partial derivative of the temporal phase with respect to time. Only a single frequency is needed for the measurement of the range-rate. Taking the partial derivative of the temporal phase with respect to time t, we obtain the range-rate of a moving target:

$$V_r = \frac{\lambda}{4\pi}\left[\frac{\partial \Phi_r}{\partial t} - \frac{\partial \Phi}{\partial t}\right], \tag{19}$$

or the temporal Doppler frequency shift :

$$f_d = \frac{1}{2\pi}\left[\frac{\partial \Phi_r}{\partial t} - \frac{\partial \Phi}{\partial t}\right], \tag{19}$$

where the second term is the range-rate glint error or Doppler glint error which causes inaccurate range-rate measurement. The range-rate glint error is given by

$$\Delta V_r = \frac{\lambda}{4\pi}\frac{\partial \Phi}{\partial t}. \tag{20}$$

We still use the two-point scatterer in Fig. 3 to illustrate the range-rate glint error.

To calculate $\frac{\partial \Phi}{\partial t}$, we know that in Eqs.(16),(12) and (13) the phase offset Φ is a function of f and α. Given a transmitted signal frequency $f = f_c$, the only parameter which could change with time is the target orientation angle α. Therefore, we can further write the partial derivative with respect to time as the following,

$$\frac{\partial \Phi}{\partial t} = \frac{\partial \Phi}{\partial \alpha}\frac{\partial \alpha}{\partial t}.$$

Then, the range-rate glint error becomes

$$\Delta V_r = \frac{\lambda}{4\pi}\frac{\partial \Phi}{\partial \alpha}\frac{\partial \alpha}{\partial t}. \tag{21}$$

Taking the partial derivative of the phase offset with respect to the angle α, we have

$$\frac{\partial \Phi}{\partial \alpha} = \frac{2\pi l \cos\alpha}{\lambda}\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2 + a_{r2}^2 + 2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha + \delta\phi)}, \tag{22}$$

and the range-rate glint error becomes

$$\Delta V_r = \frac{\lambda}{4\pi}\frac{\partial \Phi}{\partial \alpha}\frac{\partial \alpha}{\partial t} = \frac{l\cos\alpha}{2}\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2 + 2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha + \delta\phi) + a_{r2}^2}\frac{\partial \alpha}{\partial t}. \tag{23}$$

In Eq.(23), the $\partial\alpha/\partial t$ could be induced by two sources :

(a) <u>the target tangential motion</u>: from Fig. 5, we can easily see that due to target tangential motion with a cross-range velocity V_c, the target orientation angle varies with

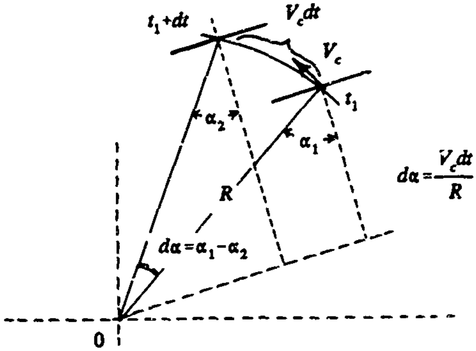

Fig.5 Target tangential motion and rotation.

time and satisfies

$$\frac{\partial \alpha}{\partial t} = \frac{r_c}{R}.$$

(b) <u>the target rotation about its geometric center</u>: if the target rotates at an angular rate ω, then

$$\frac{\partial \alpha}{\partial t} = \omega.$$

Therefore, the target tangential motion and target rotation can cause the range-rate glint error

$$\Delta V_r = \frac{l\cos\alpha}{2}\,\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2 + 2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha + \delta\phi) + a_{r2}^2}\left(\frac{V_c}{R} + \omega\right), \qquad (24)$$

or the Doppler glint error

$$\Delta f_d = \frac{l\cos\alpha}{\lambda}\,\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2 + 2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha + \delta\phi) + a_{r2}^2}\left(\frac{V_c}{R} + \omega\right), \tag{24}$$

where $l\cos\alpha$ is the tangential projection of the target length l.

Doppler glint error due to target tangential motion and rotation also exist in ISAR (inverse synthetic aperture radar) image generation. Though target motion and rotation can generate ISAR images, the tangential motion and rotation can cause a Doppler glint error which can result in distortion of the Doppler mapping of the target scatterer.

Assuming $\delta\phi = 0$, wee can see from Eq. (24) that

(a) if $\omega = 0$ and if there is no tangential motion: $V_c = 0$, then the radar is tracking the range-rate of the target geometric center;

(b) if there is no tangential motion: $V_c = 0$, but there is target rotation about its geometric center, $\omega \neq 0$, then the rotation will induce range-rate glint error;

(c) if $a_{r1} = 0$, then $\Delta V_r = (-V_c + R\omega)(l\cos\alpha)/(2R)$: the range-rate glint error is induced by the point 2 and the radar is tracking th [illegible] rate of the point 2;

(d) if $a_{r2} = 0$, then $\Delta V_r = -(-V_c + R\omega)(l\cos\alpha)/(2R)$: the range-rate glint error is induced by the point 1 and the radar is tracking the range-rate of the point 1;

(e) if $a_{r1} = a_{r2}$ and $\cos(4\pi l\sin\alpha/\lambda) \neq -1$, then $\Delta V_r = 0$: the radar is tracking the range-rate of the target geometric center;

(f) if $a_{r1} = a_{r2}$ and $\cos(4\pi l\sin\alpha/\lambda) = -1$, then $\Delta V_r = \pm\infty$: the radar is not able to track the target range-rate at all.

Fig.6 shows the range-rate glint error varying with the target orientation angle α, where we assume $f = 1$ (GHz), $l = 10$ (m), $R = 20{,}000$ (m), $V_c = 340$ (m/s), and $\omega = 0$.

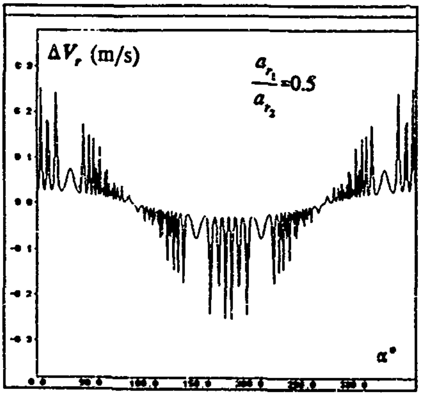

Fig.6 The range-rate glint error varies with target orientation angles.

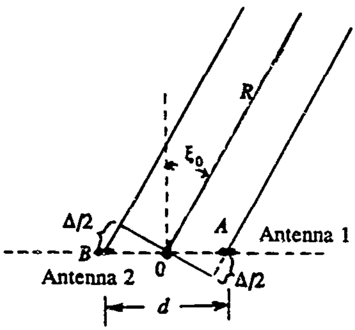

Fig.7 A simple interferometer.

4. Angle Measurement and Angle Glint Error

The measurement of the angle of arrival requires spatial phase or spatial phase spectrum. Fig.7 shows the geometry of a simple interferometer for measurement of angle of arrival.

The transmitter at the position O transmits a signal. The returned signal at the atenna 1 from the target is

$$s_1(t) = a_r e^{j2\pi f(t-T_1)},$$

where $T_1 = (2R-\Delta/2)/c$ is the time delay at the position A, $\Delta/2 = (d/2)\sin\xi_0$, and ξ_0 is the angle of arrival.

The returned signal at the antenna 2 from the same target is

$$s_2(t) = a_r e^{j2\pi f(t-T_2)},$$

where $T_2 = (2R+\Delta/2)/c$ is the time delay at the position B.

The phase difference between the two antennas is the spatial phase, represented as

$$\Phi_{1,2} = 2\pi f(T_2-T_1) = 2\pi f\frac{d\sin\xi_0}{c} = \frac{2\pi}{\lambda}d\sin\xi_0. \tag{25}$$

To resolve angle ambiguity, we could use (1) multiple position: d_1, d_2, ... d_M or (2) multiple frequency. By taking the partial derivative of the spatial phase with respect to spatial position d, or taking the partial derivative of the spatial phase spectrum with respect to frequency, the angle of arrival ξ_0 can be extracted

$$\sin\xi_0 = \frac{\lambda}{2\pi}\frac{\partial\Phi_{1,2}}{\partial d}, \tag{26}$$

or

$$\sin\xi_0 = \frac{c}{2\pi d}\frac{\partial\tilde{\Phi}_{1,2}}{\partial f}. \tag{26}$$

However, Eq.(25) is accurate only for the case of a point scatterer. When dealing with a distributed scatterer, we must modify Eq.(25) by adding range glint errors at antenna 1 and 2.

We still use a two-point scatterer as an example. For the two-point scatterer, the phase difference between the two antennas becomes

$$\Phi_{1,2} = 2\pi f(T_2-T_1) = \frac{2\pi}{\lambda}d\sin\xi+(\Phi_2-\Phi_1), \tag{27}$$

where Φ_1 is the phase offset at antenna 1 caused by the distributed scatterer, and Φ_2 is the phase offset at antenna 2 caused by the same reason. Φ_1 and Φ_2 are given by Eq.(16). Therefore, the angle of arrival becomes

$$\sin\xi = \frac{\lambda}{2\pi}\frac{\partial\Phi_{1,2}}{\partial d}+\frac{\lambda}{2\pi}[\frac{\partial\Phi_2}{\partial d}-\frac{\partial\Phi_1}{\partial d}]. \tag{28}$$

As shown in Fig.8, the target orientation angle from the antenna 1 is α_1 and from the antenna 2 is α_2. Angles α_1 and α_2 can also be expressed in terms of the target orientation angle from the center of the interferometer α. By geometric calculation we derive the relationship between α and α_1 , and the relationship between α and α_2 as the following

$$\alpha_1 = \alpha-\frac{d\cos\xi_0}{2R} \tag{29}$$

and

$$\alpha_2 = \alpha+\frac{d\cos\xi_0}{2R}. \tag{30}$$

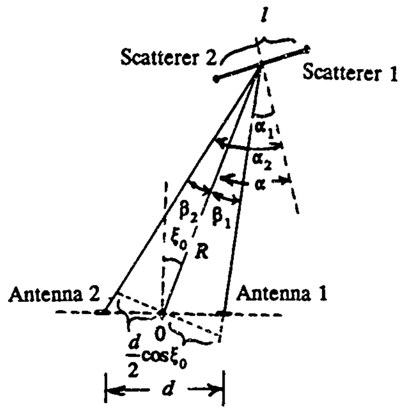

Fig.8 The angle of arrival for a two-point scatterer.

To analyze the angle glint error, we need to calculate $\partial\Phi_1/\partial d$ and $\partial\Phi_2/\partial d$. From Eqs.(16) and (29), for antenna 1 we have

$$\Phi_1 = \arctan\frac{a_{r1}\sin\Psi_{11}+a_{r2}\sin\Psi_{12}}{a_{r2}\cos\Psi_{11}+a_{r2}\cos\Psi_{12}}, \tag{31}$$

where

$$\Psi_{11} = \frac{2\pi}{\lambda}l\sin\alpha_1+\phi_{11} = \frac{2\pi}{\lambda}l\sin\left(\alpha-\frac{d\cos\xi_0}{2R}\right)+\phi_{11}, \tag{32}$$

and

$$\Psi_{12} = \frac{2\pi}{\lambda}l\sin\alpha_1+\phi_{12} = \frac{2\pi}{\lambda}l\sin\left(\alpha-\frac{d\cos\xi_0}{2R}\right)+\phi_{12}, \tag{33}$$

where ϕ_{11} is the reflected phase change at point scatterer 1 to antenna 1, and ϕ_{12} is the reflected phase change at point scatterer 2 to antenna 1.

For antenna 2 we also have

$$\Phi_2 = \arctan\frac{a_{r1}\sin\Psi_{21}+a_{r2}\sin\Psi_{22}}{a_{r2}\cos\Psi_{21}+a_{r2}\cos\Psi_{22}}, \tag{34}$$

where

$$\Psi_{21} = \frac{2\pi}{\lambda}l\sin\alpha_2+\phi_{21} = \frac{2\pi}{\lambda}l\sin(\alpha+\frac{d\cos\xi_0}{2R})+\phi_{21}, \tag{35}$$

and

$$\Psi_{22} = \frac{2\pi}{\lambda}l\sin\alpha_2+\phi_{22} = \frac{2\pi}{\lambda}l\sin(\alpha+\frac{d\cos\xi_0}{2R})+\phi_{22}, \tag{36}$$

where ϕ_{21} is the reflected phase change at point scatterer 1 to antenna 2, and ϕ_{22} is the reflected phase change at point scatterer 2 to antenna 2.

Using the result derived earlier in Eq.(22), we have

$$\frac{\partial\Phi_1}{\partial d} = \frac{\partial\Phi_1}{\partial\alpha}\frac{\partial\alpha}{\partial d} = \frac{2\pi l\cos\alpha}{\lambda}\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2+a_{r2}^2+2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha+\delta\phi_1)}\frac{\cos\xi_0}{2R}, \tag{37}$$

and

$$\frac{\partial\Phi_2}{\partial d} = \frac{\partial\Phi_2}{\partial\alpha}\frac{\partial\alpha}{\partial d} = \frac{2\pi l\cos\alpha}{\lambda}\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2+a_{r2}^2+2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha+\delta\phi_2)}\frac{-\cos\xi_0}{2R}, \tag{38}$$

where $\delta\phi_1 = \phi_{12}-\phi_{11}$ and $\delta\phi_2 = \phi_{22}-\phi_{21}$.

Substituting Eq.(37) and (38) into the second term of Eq.(28), and assuming $\delta\phi_1 = \delta\phi_2 = \delta\phi$, we have

$$\frac{\lambda}{2\pi}\left[\frac{\partial\Phi_2}{\partial d}-\frac{\partial\Phi_1}{\partial d}\right] = l\cos\alpha\frac{a_{r2}^2 - a_{rl}^2}{a_{rl}^2+a_{r2}^2+2a_{rl}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha+\delta\phi)}\frac{\cos\xi_0}{R}. \tag{39}$$

On the other hand, because of the error $\Delta\xi$, the angle of arrival becomes

$$\xi = \xi_0+\Delta\xi.$$

Applying the Taylor expansion to $\sin\xi$, we have

$$\sin\xi = \sin(\xi_0+\Delta\xi) = \sin\xi_0+\cos\xi_0\Delta\xi-\frac{1}{2!}\sin\xi_0(\Delta\xi)^2+... \tag{40}$$

The first term in Eq.(40) corresponds to the first term of the right hand side of Eq.(28), which is the measured value of the angle of arrival. The second term in Eq.(40) is the first order error, which corresponds to the second term of the right hand side of Eq.(28). Therefore, we obtained the angle glint error

$$\Delta\xi = \frac{l\cos\alpha}{R}\frac{a_{r2}^2 - a_{rl}^2}{a_{rl}^2+a_{r2}^2+2a_{rl}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha+\delta\phi)}, \tag{41}$$

where $l\cos\alpha/R$ is the angle occupied by the target length l.

Assuming $\delta\phi = 0$, we can see from Eq.(41) that

(a) if $a_{rl} = 0$, then $\Delta\xi = l\cos\alpha/R$: the angle glint error is induced by the point 2 and the radar is tracking the point 2;

(b) if $a_{r2} = 0$, then $\Delta\xi = -l\cos\alpha/R$: the angle glint error is induced by the point 1 and the radar is tracking the point 1;

(c) if $a_{rl} = a_{r2}$ and $\cos(4\pi l\sin\alpha/\lambda) \neq -1$, or $\alpha = \pm\pi/2$, then $\Delta\xi = 0$: the radar is tracking the angle of the target geometric center;

(d) if $a_{rl} = a_{r2}$ and $\cos(4\pi l\sin\alpha/\lambda) = -1$, then $\Delta\xi = \pm\infty$: the radar is not able to track the target angle at all.

Fig. 9 shows the angle glint error varying with the target orientation angle α, where we assume $f = 1$ (GHz), $R = 20{,}000$ (m), and $l = 10$ (m).

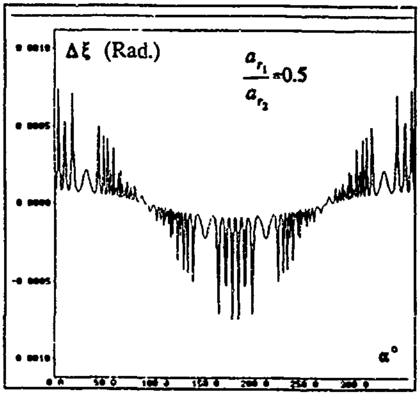

Fig.9 The angle glint error varies with target orientation angles.

5. Angle-Rate Measurement and Angle-Rate Glint Error

The angle-rate can be extracted from partial derivative of the spatial phase with respect to time. Only a fixed spatial position d is needed for the measurement of the angle-rate. By taking the partial derivative of the spatial phase with respect to time t, we obtain the angle-rate of a moving target :

$$\omega_T = \frac{d\xi}{dt} = \frac{\lambda}{2\pi d\cos\xi_0}\left[\frac{\partial\Phi_{12}}{\partial t} + \left(\frac{\partial\Phi_2}{\partial t} - \frac{\partial\Phi_1}{\partial t}\right)\right], \tag{42}$$

or the spatial Doppler frequency shift of the moving target is

$$f_s = \frac{1}{2\pi\cos\xi_0}\left[\frac{\partial\Phi_{1,2}}{\partial t} + \left(\frac{\partial\Phi_2}{\partial t} - \frac{\partial\Phi_1}{\partial t}\right)\right], \tag{42}$$

where the second term is the angle-rate glint error caused by the distributed scatterer. Due to the angle-rate glint error, the measurement of angle-rate is not accurate. The angle-rate glint error is

$$\Delta\omega_T = \frac{\lambda}{2\pi d\cos\xi_0}\left[\frac{\partial\Phi_2}{\partial t} - \frac{\partial\Phi_1}{\partial t}\right]. \tag{43}$$

We still use the two-point scatterer to illustrate the angle-rate glint error. For a given transmitted signal frequency and for a given antenna spatial position, the only parameter which varies with time is the target orientation angle α_1, α_2. Therefore, we can further write the partial derivative of spatial phases with respect to time as the following,

$$\frac{\partial\Phi_1}{\partial t} = \frac{\partial\Phi_1}{\partial\alpha}\frac{\partial\alpha}{\partial t} = \frac{\partial\Phi_1}{\partial\alpha}\frac{\partial\left(\alpha_1 + \frac{d\cos\xi_0}{2R}\right)}{\partial t} = \frac{\partial\Phi_1}{\partial\alpha}\frac{\partial\alpha_1}{\partial t} \tag{44}$$

and

$$\frac{\partial\Phi_2}{\partial t} = \frac{\partial\Phi_2}{\partial\alpha}\frac{\partial\alpha}{\partial t} = \frac{\partial\Phi_2}{\partial\alpha}\frac{\partial\left(\alpha_2 - \frac{d\cos\xi_0}{2R}\right)}{\partial t} = \frac{\partial\Phi_2}{\partial\alpha}\frac{\partial\alpha_2}{\partial t}. \tag{45}$$

Then, the angle-rate glint error becomes

$$\Delta\omega_T = \frac{\lambda}{2\pi d\cos\xi_0}\left[\frac{\partial\Phi_2}{\partial\alpha}\frac{\partial\alpha_2}{\partial t} - \frac{\partial\Phi_1}{\partial\alpha}\frac{\partial\alpha_1}{\partial t}\right]. \tag{46}$$

Again, using the result in Eq.(22) and assuming $\delta\phi_1 = \delta\phi_2 = \delta\phi$, we obtain

$$\left[\frac{\partial\Phi_2}{\partial\alpha}\frac{\partial\alpha_2}{\partial t} - \frac{\partial\Phi_1}{\partial\alpha}\frac{\partial\alpha_1}{\partial t}\right] = \frac{2\pi l\cos\alpha}{\lambda}\,\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2 + a_{r2}^2 + 2a_{r1}a_{r2}\cos\left(4\pi\frac{l}{\lambda}\sin\alpha + \delta\phi\right)}\,\frac{\partial}{\partial t}(\delta\alpha), \tag{47}$$

and the angle-rate glint error becomes

$$\Delta\omega_T = \frac{l\cos\alpha}{d\cos\xi_0}\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2+2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha+\delta\phi)+a_{r2}^2}\frac{\partial}{\partial t}(\delta\alpha), \tag{48}$$

where $\delta\alpha = \alpha_2-\alpha_1$.

$\partial(\alpha_2-\alpha_1)/\partial t$ is induced by the target tangential motion. Due to target tangential motion the difference of the two orientation angles varies. The target rotation does not affect the rate of change of $(\alpha_2-\alpha_1)$ if there is no target tangential motion.

Given a target range R, angle ξ_0, and a target cross-range velocity V_c, from Fig.10 we know

$$\delta\alpha = \alpha_2-\alpha_1 = \frac{d\cos\xi_0}{R}. \tag{49}$$

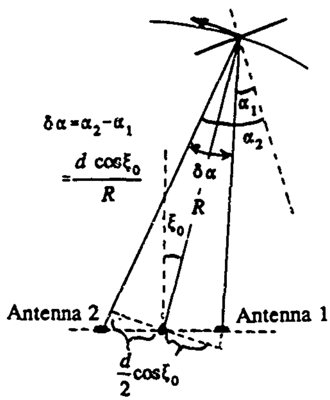

Fig.10 The geometric relationship between α_1 and α_2.

Therefore, by taking the partial derivative with respect to t, we obtain

$$\frac{\partial}{\partial t}(\delta\alpha) = \frac{\partial}{\partial t}(\alpha_2-\alpha_1) = \frac{V_c}{R}\frac{d\sin\xi}{R}. \tag{50}$$

Substituting Eq.(50) into Eq.(48), we have the angle-rate glint error, represented as

$$\Delta\omega_T = \frac{l\cos\alpha V_c \tan\xi_0}{R^2}\frac{a_{r2}^2 - a_{r1}^2}{a_{r1}^2+2a_{r1}a_{r2}\cos(4\pi\frac{l}{\lambda}\sin\alpha+\delta\phi)+a_{r2}^2}. \tag{51}$$

Eq.(50) tells us that for a given range R and a given cross-range velocity V_c, $\partial(\alpha_2-\alpha_1)/\partial t$ varies with the target angle ξ_0.

Assuming $\delta\phi = 0$, we can see from Eq.(51) that

(a) if $a_{r1} = 0$ and $\cos\alpha V_c \tan\xi_0 \neq 0$, then $\Delta\omega_T = \frac{l\cos\alpha V_c\tan\xi_0}{R^2}$: the angle-rate glint error is induced by the point 2;

(b) if $a_{r2} = 0$ and $\cos\alpha V_c \tan\xi_0 \neq 0$, then $\Delta\omega_T = -\frac{l\cos\alpha V_c\tan\xi_0}{R^2}$ the angle-rate glint error is induced by the point 1;

(c) if $a_{r1} = a_{r2}$ and $\cos(4\pi l\sin\alpha/\lambda) \neq -1$, $\cos\alpha V_c\tan\xi_0 \neq 0$, then $\Delta\omega_T = 0$: the radar is tracking the angle-rate of the target geometric center;

(d) if $a_{r1} = a_{r2}$ and $\cos(4\pi l\sin\alpha/\lambda) = -1$, $\cos\alpha V_c\tan\xi_0 \neq 0$, then $\Delta\omega_T = \pm\infty$: the radar is not able to track the target angle-rate at all;

(e) if $V_c = 0$ or $\xi_0 = 0$ or $\alpha = \pm\pi/2$, then $\Delta\omega_T = 0$: the radar is tracking the angle-rate of the target geometric center;

(f) if $\xi_e \to \pm\pi/2$ and $\cos\alpha V_c \neq 0$, then $\Delta\omega_T \to \pm\infty$: the radar is not able to track the target angle-rate at all.

Fig.11 shows the angle-rate glint error varying with the target orientation angle α, where we assume $f = 1$ (GHz), $R = 20{,}000$ (m), $l = 10$ (m), $V_c = 340$ (m/s), and $\xi_0 = \pi/4$.

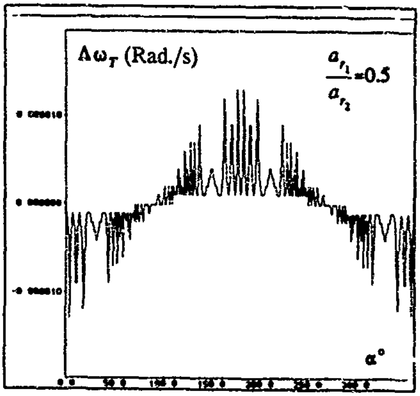

Fig.11 The angle-rate glint error varies with α.

6. Summary

As shown in [1,2,3], the echo signal phase provides information on range, range-rate, angle, and angle-rate. In this report, we described how to measure the information from the partial derivatives of the echo signal phase from a distributed scatterer. Unlike measuring target information from a point scatterer, for a distributed scatterer we must take account of glint errors caused by the dimension and the orientation of the distributed scatterer. For simplicity, in this report we used a two-point scatterer as an example to calculate glint errors. The calculation and the result can be generalized to a multi-point scatterer.

In the case of a distributed scatterer, the range information can be measured from the partial derivative of the phase with respect to frequency. To resolve range ambiguities, multiple frequencies must be used. The measured range of the distributed scatterer includes a range glint error term

$$\Delta R = \frac{c}{4\pi}\frac{\partial\Phi}{\partial f} = \frac{l\sin\alpha}{2}\frac{1-\rho^2}{1+\rho^2+2\rho\cos(4\pi\frac{l}{\lambda}\sin\alpha)},$$

$$(\ \rho = a_{r1}^2/a_{r2}^2; \quad \delta\phi = 0\ )$$

which is related to the following factors :

(1) the wavelength of the transmitted signal;
(2) the ratio of the amplitudes of the individual scatterers;
(3) the radial projection of the target length, determined by the orientation of and the length of the target; and
(4) the difference bet `en the phases induced by each scatterer.

The orientation of the target, which is usually unpredictable, will affect the glint error. There is no range glint error if the connecting line between the two scatterers (the orientation of the target) is perpendicular to the radial line between the radar and the target. If the ratio of the amplitudes of the individual scatterers is one, there is also no range glint error. However, in general, range glint error does exist. For a given wavelength and a given target size, the range glint error varies with the orientation of the target. For some particular orientations of the target, the range glint error can be too large to track the range.

The range-rate information can be measured from the partial derivative of the phase with respect to time. The range-rate glint error term is

$$\Delta V_r = \frac{l\cos\alpha}{2}\frac{1-\rho^2}{1+\rho^2+2\rho\cos(4\pi\frac{l}{\lambda}\sin\alpha)}(\frac{V_t}{R}+\omega).$$

$$(\ \rho = a_{r1}^2/a_{r2}^2; \quad \delta\phi = 0\ )$$

In addition to the factors which affect the range glint error mentioned above, the range-rate glint error is also related to the tangential projection of the target length, the target rotation-rate about its geometric center, and the velocity of the target tangential motion. If there is no tangential motion and rotation, there will be no range-rate glint error. However, when the tangential motion or the rotation exists, for some particular orientations, the range-rate glint error could be so large that the radar is unable to track the range-rate at all. The range-rate glint error due to tangential motion and rotation also exist in ISAR. Although the target motion and rotation can generate ISAR images, the tangential motion and rotation could also cause glint errors which can result in distortion of the Doppler mapping of the target scatterers.

The angular information can be extracted from echo signal phase by making use of more than one antenna element; such as an interferometer with two separated elements. To resolve angle ambiguities, multiple positions of the elements or multiple frequencies may be used. By taking the partial derivative of the phase with respect to position, or taking the partial derivative of the phase spectrum with respect to frequency, the angle of arrival can be measured. The angle glint error is

$$\Delta\xi = \frac{l\cos\alpha}{R}\frac{1-\rho^2}{1+\rho^2+2\rho\cos(4\pi\frac{l}{\lambda}\sin\alpha)},$$

$$(\ \rho = a_{r1}^2/a_{r2}^2; \quad \delta\phi = 0\ )$$

which is related to the following factors:

1) the wavelength of the transmitted signal;
2) the ratio of the amplitudes of the individual scatterers;
3) the radial projection of the target length, determined by the orientation and the length of the target;
4) the difference between the phases induced by each scatterer; and
5) the angle occupied by the target dimension, which is the ratio of the tangential projection of the target length to the target range.

When the orientation of the target coincides with the radial line between the radar and the target, there will be no angle glint error at all. For some particular orientations of the target, the angle glint error can be too large for radar to track the angle.

The angle-rate is measured from the partial derivative of the phase with respect to time. The angle-rate glint error term is

$$\Delta\omega_T = \frac{l\cos\alpha V_c\tan\xi_0}{R^2}\frac{1-\rho^2}{1+\rho^2+2\rho\cos(4\pi\frac{l}{\lambda}\sin\alpha)}.$$

$$(\ \rho = a_{r1}^2/a_{r2}^2; \quad \delta\phi = 0\ )$$

In addition to the factors which affect the angle glint error mentioned above, the angle-rate glint error is also related to the target angle of arrival, the target cross-range velocity, and the target range. When the orientation of the target coincides with the radial line between the radar and the target, there will be no angle-rate glint error if target angle of arrival is not very low. Otherwise, for very low angle of arrival or for some particular orientations of the target, the angle-rate glint error can be such large that the radar is unable to track the angle-rate at all.

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